The Remembrance of Problems and of their Solutions

A STUDY IN LOGICAL MEMORY

BY

E. O. FINKENBINDER

A DISSERTATION SUBMITTED TO THE FACULTY OF CLARK UNIVERSITY, WORCESTER, MASS., IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY, AND ACCEPTED ON THE RECOMMENDATION OF JOHN WALLACE BAIRD

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A STUDY IN LOGICAL MEMORY[1]

By E. O. FINKENBINDER

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I. INTRODUCTION

The investigation which is here reported has aimed to determine what mental procedures and what mental processes are employed in solving problems of various sorts, and in subsequently recalling the problems and the solutions. Problems of various kinds and of various degrees of difficulty and complexity,—mathematical problems, mechanical puzzles, and the like,—were presented for solution; and after the lapse of a month or more the observer attempted to recall both the problem and his solution of it. At the time of the initial act of solving and also of the subsequent act of recall, he furnished an introspective description of the mental content and the mental procedure to which he had had recourse. It was hoped by this means to throw light upon the processes of understand-

[1] From the Psychological Laboratory of Clark University.

ing and solving problems; but the chief purpose of the investigation consisted in obtaining an insight into the mental mechanism which is employed in the remembering of data which are logically related.

II. Historical Statement

Numerous investigators have attacked the problem of 'mechanical memory,' but the act of remembering logically related data has received but slight attention. The pioneer experiments of Ebbinghaus [2] were concerned almost exclusively with the learning and the forgetting of nonsense-syllables. It is true that he also employed significant material,—stanzas of poetry,—but he was interested only in the quantitative aspects of his problem, and he made no attempt to furnish a qualitative analysis of the phenomena of memory. And the work of Radossawljewitsch [3] suffers from the same limitation. Although Henderson [4] was interested chiefly in the quantitative relations of memorial phenomena his paper contains a section which deals with the qualitative analysis of the process of remembering. He finds that three factors contribute to the act of recall: a preliminary adjustment of attention, which is comparable to that which was present during the initial act of learning, and which is accompanied by images of that initial experience; a sense of the general meaning of the material which was learned; and an unfolding of details which is a product either of one's waiting for certain cues to lead to results, or of one's inferring the details from the cues and subsequently employing the criterion of familiarity as a test of the validity of the inference.

In a more recent investigation, Michotte and Ransy [5] presented pairs of related words to their observers and after an interval the first word of a pair was re-presented with the instruction that its mate was to be recalled. It was found that this act of reproduction was usually accomplished by means of an intermediate term: a visual image of the object denoted by the word which was sought; a visual image of the posi-

[2] H. Ebbinghaus. *Ueber das Gedächtnis: Untersuchungen zur experimentellen Psychologie.* Leipzig, Duncker und Humblot, 1885. Pp. 188.

[3] Paul R. Radossawljewitsch, *Das Behalten und Vergessen bei Kindern und Erwachsenen.* Leipzig, Nemnich, 1901. Pp. 197.

[4] E. N. Henderson. A Study of Memory for Connected Trains of Thought. *Psy. Rev. Monog. Suppl.* No. 23. IV. 1903. Pp. 94.

[5] A. Michotte and C. Ransy. Contribution à l'étude de la mémoire logique. *Travail du laboratoire de psychologie expérimentale de l'université de Louvain.* Louvain, 1912. pp. 95.

tion of the word upon the printed sheet; a motor image; an affective process; or a consciousness of the relation which obtains between the stimulus-word and its mate. This relationship may be symbolized by an image, or it may make its appearance in non-imaginal form. The authors believe that imagery plays only a secondary rôle; the image fails to represent the relation in an adequate fashion. The authors point out that the relation is the essential factor in the act of recall; but the introspections of certain of their observers seem to show that the presence of the imaginal symbol ensures the recall of the relation.

Bühler's[6] investigation of the remembering of 'thoughts' yielded a somewhat similar result. His method consisted in first presenting groups of sentences or phrases to his observers, and subsequently in presenting analogous (but non-identical) sentences or phrases. The observer reported what sentences or phrases of the original group were evoked by these cues and described the mental processes which were involved in the act of reproduction.[7] Bühler's findings led him to posit the existence of a non-imaginal consciousness of relation; the recall of the complete meaning of a sentence may be mediated by a pure 'thought' without the coöperation of imagery of any sort.

Hirszowicz[8] investigated the processes which attend the act of free association upon the presentation of adjectives and nouns, which appealed to all sense departments. She found that when the stimulus-word is an adjective, merely the quality may be first imagined, then the object to which it belongs; as, for example, first a yellow color with spread-out surface was seen, not connected with an object but very bright, then the surface seemed to be rounded and a lemon appeared. For one observer the images came very rapidly and developed into a definite visual image. Kinaesthetic experiences were sometimes localized in parts of the body; but these and organic experiences were very rare. Definite feeling tone was noted in about forty per cent. of all the reactions. The feeling tone arose from the visual image of the stimulus or reaction word;

[6] Karl Bühler. Tatsachen und Probleme zu einer Psychologie der Denkvorgänge. III. Ueber Gedankenerinnerungen. *Archiv f. d. ges. Psychol.* XII. 1908. 24-92.

[7] It should be noted in this connection that Bühler's published introspections are selected almost exclusively from two of his six observers.

[8] Sarah S. Hirszowicz. Experimentelle Beiträge zur Analyse des Reproductionsvorgangs. (*Inaug. diss.*) Zürich: Selnau, Leemann & Cie., 1909. pp. 71.

as, for example, the stimulus word was salad and the visual image of a salad with green leaves appeared, accompanied by a feeling of pleasure. Often the stimulus word aroused at first a memory complex; as, for instance, a memory of a definite poem, and with it a feeling of pleasure. The emotional states in the course of reproduction play a much greater rôle in women than in men.

Schwiete's [9] investigation of the conscious structure of concepts is related to our problem in that he employed the method of reproduction to establish both the free and directed associations. Words belonging to the various 'parts of speech' were given as stimuli. In a majority of the reactions visual images of objects associated with the stimulus word were aroused. In certain cases whole constellations of ideas appeared which embodied several applications of the concept, or several appropriate conditions. The meanings, or more complete relations were not seen at once but were developed with the constellations. The structure of concepts evoked in recall are, for the most part, images, and 'feelings of familiarity.' These images are generally dim and indefinite, but at times complete visual images do appear.

III. Our Experimental Arrangement

A. *Observers*

The eighteen observers named below took part in this investigation. Dr. J. W. Baird, (*B*); Dr. A. K. Beik, (*Bk*); Dr. L. A. Blue, (*Bl*); Mr. Walter Butler, A. M., (*Bt*); Mr. P. C. Chang, (*C*); Dr. Helen D. Cook, (*Ck*); Mrs. E. O. Finkenbinder, A. B., (*F*); Dr. S. W. Fernberger, (*Fr*); Dr. S. C. Fisher, (*Fs*); Dr. A. N. Gilbertson, (*G*); Miss B. W. Higgins, (*H*); Dr. J. Allen Hunter, (*Hn*); Mr. Douglas Miner, A. B., (*M*); Dr. Roy F. Richardson, (*R*); Dr. Elizabeth Woods, (*W*); Mr. Raymond M. Wheeler, A. M., (*Wh*); Dr. H. P. Weld, (*Wld*); Dr. S. Yamada, (*Y*).

B. *Method and Arrangement of the Experiments*

From February to June, 1912, problems were presented for solution to ten observers, one observer at a time and in a quiet room; and recalls were made one month or more after solving. Each observer made a second recall of each problem at the close of the year (one month or more after the first recall); and after the vacation of three months, a third recall. The

[9] F. Schwiete. Ueber die psychische Repräsentation der Begriffe. *Archiv f. d. ges. Psychol.*, XIX., 1910, 475-544.

observers who participated were:—*Bk, Bl, Bt, C, Ck, Fs, Hn, M, Wh, Wld.*

A second series of problems were solved during the year 1912-13 by the following ten observers: *B, F, Fr, Fs, G, H, R, W, Wh, Y.* The problems were not presented in the same order to more than two observers,—the changed order being introduced for the purpose of equalizing the possible influence of one problem upon another. For example, if under this irregular order, all observers in attempting to recall a certain problem, recall first the problem that preceded it in the series, we may be sure that this course of association in recall is a general phenomenon. If, however, this course held true for two problems that had been presented in the same order for all observers, it might still be due to inherent qualities of the problems and not to the relative position of the two in our order of presentation.

The materials were presented in either a visual or an auditory fashion. Those statements of problems that were read by the observers were either typewritten, or written in the experimenter's hand-writing. Our interest here was to determine whether the course of recall is the same for the three modes of presentation. (See p. 72.)

The observers were instructed, in certain cases, to read the statement carefully, then to introspect upon the perception of the problem up to that point. In other cases, they were allowed to proceed with the solution without introspecting upon the initial act of apprehending the problem. When the observer had made a step or a number of steps,—which fact was apparent to the experimenter from the observer's actions, statements, or penciled notes,—the experimenter interrupted with the request, " Introspect," or " Introspect upon your last step." It frequently happened that the observer had become so deeply interested in the solving that he turned with reluctance to introspect. On the other hand, at times his attention turned immediately to the introspection, and he began his description voluntarily. The reader may object here that in thus introducing interruptions, we have interfered with the observer's natural procedure in the solving of the problems. But if the solving be carried through from beginning to end without a break, the mental processes are often so numerous that a complete introspection of the whole series at once is impossible. We are interested in determining just what processes occur in the solving, in order that we may determine in the subsequent recall what proportion of the processes are recurrences of the steps in the initial solving. Without a de-

tailed knowledge of the processes occurring in the solving we can not determine what is recalled, and this method of introspection seems most appropriate.

The uninterrupted introspective report was written as dictated by the observer; then questions concerning unclear description of structure and of function were asked by the experimenter. These questions were also recorded in every instance. In addition to the recording of introspective reports and questions, we noted the remarks, gestures, or other expression of the observer, as well as the successive steps pursued in the solution. In the case of one observer, *Wh,* plethysmographic and pneumographic records were obtained; these, however, contributed little or nothing to our investigation, and they will not be described or discussed in our statement of results.

It turned out that certain observers failed to solve some of the problems assigned. In such cases, the experimenter furnished the solution after an hour had been spent in an unsuccessful attempt.

The observers were instructed not to attempt a recall of the problems at any time between sittings; they were also instructed to make an effort to direct their attention elsewhere in case the problem began to intrude itself into consciousness during the intervals between sittings.

In the reproduction of the initial group of ten problems, the observer was first asked to recall the first problem that he had solved; and at each subsequent sitting, the problem next in the order of solving. This method affords information concerning the particular factors by which the problems were ushered again into consciousness, in that the experimenter presented no datum of any problem to the observer as a clue to recall. If in his act of recall, the observer hit upon a problem, which had been presented later than the one called for, he was not informed of his error, but was allowed to proceed with a complete recall of that problem, and of its solution. If he were unable to recall the problem or its solution in detail, he was given certain cues, or was asked questions which directed his attention to various points that had been overlooked. These cues and questions were recorded, as were all other questions asked by the experimenter.

In the later experiments, cues were given for the beginning of a recall, as, for example, "Recall the density problem." (During the solving, the name "density" had not been given to any problem, although several involved that quality of matter.) Cues both of an abstract and a concrete nature were given, in order to enable us to determine their relative value

in the act of recalling the problems to consciousness. (See p. 70.)

The method of recording the results in recall was the same as in the initial solving, as already described, except that when no farther recall seemed to be forthcoming, additional cues were given. The observer was encouraged to guess and to fill up the gaps in his recall; and this frequently led to a statement of the problem or its solution. When he wholly failed to recall the problem, it was presented for re-solution. This method furnishes data for a comparison of the courses taken in the two solutions.

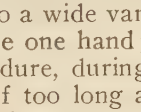
It will be found that our problems are more difficult than those which have been employed by any other investigator of this topic. It will be found, too, that the intervals which elapse between the act of learning and the act of recalling, in our investigations, are relatively long. And our method differs from that of former investigators in that we have made an attempt to discover the structure as well as the significance or function of the mental processes which occur in the act of solving problems. Every act of memory is concerned, of course, with a mental content which has been acquired at some previous time; and in any investigation of memory, it is essential to determine just what was present to consciousness during this initial act of acquisition; without this datum it is obviously impossible to institute any comparison between original experience and subsequent recall.

C. *Materials*

Our materials consisted of thirty-two 'problems' which had been selected as appropriate for our investigation. They gave rise to a wide variety of mental procedure and mental content, on the one hand; and on the other hand, the steps in mental procedure, during solving and during subsequent recall, were not of too long a duration to admit of detailed introspective analysis.

These problems as quoted below were presented to our observers; the name which appears here within parentheses is appended for later reference in our exposition.

1. (*The Dot Problem*)



"Readjust the lines so as to make groups of 6, 6, and 4, instead of the present four groups of 8, 3, 3, and 2. You must move only two of the lines in order to attain this end."

2. (*The* T *Problem*)

"Assemble these pieces of cardboard to form a capital T."

3. (*The Ring Problem*)

This mechanical puzzle consisted of a small hexagonal plate of vulcanite, and a ring of heavy wire. The plate was pierced by seven holes, and two additional holes had been bored half-way through it. The two ends of the wire which constituted the ring were not exactly opposed, but were slightly displaced in a lateral direction. The plate, with the ring passing through one of the holes was put into the hands of the observer with the instructions that he should remove the ring from the plate. The problem could be solved very simply by rotating the ring, threading it through several holes in succession until it was finally passed through the central hole, where by tilting the ring, it could be detached from the plate.

4. (*The Touch Problem*)

A sheet of paper was laid upon the observer's forehead; two or more dots were then made upon the paper. The observer was then asked to transcribe these dots, in correct order and position, upon a sheet of paper which lay before him on the table.

5. (*The Memorizing Problem*)

"You will be given one half-minute to memorize the numbers on this sheet."

8 22
15 29 1
26 47 62
4
33 100

The page of script figures was presented to the observer and after he had memorized for one half-minute he was called upon for his general method of procedure. Then a second half-minute was spent in memorizing, after which a complete introspective account was given.

6. (*The Memorizing Problem—Division*)

A sheet containing the following group of numbers was presented with the same instructions as in Problem 5. The second half-minute was spent in noting the numerical relations and in memorizing.

```
          5
  27   16     7
         21
12 13    48
         18
 19   10
```

(Problems 5 and 6 were designed to bring to light the differences between the recall of simple, isolated data and the recall of numerically related data.)

7. (*The Addition Problem*)

" Choose three of these numbers whose sum is 50."

```
          3
        25 27
      15  6  14
   43  9  31  19
```

8. (*The Counters Problem*)

Five cards, each 5 cm. by 7.5 cm., labelled A, B, C, D, E, and twenty counters 2.5 cm. in diameter, numbered consecutively from one to twenty, were presented with the following instructions: " Place the five cards on the table, A to your left and away from you, B about 5 cm. to its right, C midway between the two but nearer to you by the length of a card, D to your left with its furthermost corner at the right near the closer left corner of C, and put E to the right of C as D is to the left. Place the ten counters in numerical order on C, one at the top and ten at the bottom. You are required to move the complete pile of counters to another card, one at a time, never placing any counter on one bearing a smaller number than itself. *a.* What is the fewest number of moves necessary?"

b. " In how few moves may a pile of counters numbered three to thirteen inclusive be moved to a second card in like manner?"

c. " In how few moves can a pile of twenty counters be placed on a second card in the same manner?"

d. "In how few moves can a pile of five counters be placed on a second card in the same manner?"

e. "In how few moves can a pile of fifty counters be placed on a second card in the same manner?"

9. (*The Box Problem*)

"What are the dimensions of a rectangular box whose top contains 120 square cm., whose side contains 96 square cm., and whose end contains 80 square cm.?"

10. (*The Weight Problem*)

"A man had a forty-pound weight which he broke into pieces of such sizes that they enabled him to weigh any number of pounds from 1 to 40. What did the pieces severally weigh?"

11. (*The Hog Problem*)

"A farmer, when asked the weight of a certain hog, replied, 'Just seven score pounds and a half its weight.' How much did the hog weigh?"

12. (*The Spider Problem*)

"A spider is on the end-wall of a room, one meter from the ceiling, and five meters from each side. The room is 10 meters high and 20 meters long. A fly is one meter from the floor and five meters from each side, on the end of the room opposite the spider. The spider crawls on the surface of the room to the spot where the fly is. What is the shortest route? It is less than 30 meters."

13. (*The Triangle Problem*)

"Outline[10] all the kinds of triangles you know."

14. (*The Number Arrangement Problem*)

$99\ 9/9 = 100$; $(5+5) \times (5+5) = 100$. Find other similar combinations of digits that equal 100."

15. (*The Hat Problem*)

"In how many different ways may two men exchange hats? Three men? Four men? Five men? Six men? Seven men? Eight men? etc. Find a rule for determining the number."

[10] The word "outline" was here employed ambiguously to permit the observer to solve the problem according to either or both meanings. Our interest was to determine how the meanings are recalled.

16. (*The Multiplication Problem*)

"Suppose in a multiplication problem the nine digits are used in multiplicand, multiplier, and quotient, *i. e.*, four in the multiplicand, one in the multiplier, and the remaining four of the nine in the quotient. Choose the numbers for multiplicand and for multiplier so that the quotient will be the four digits that you have not used in the multiplicand and multiplier."

17. (*The Apple Problem*)

"I paid a shilling for some apples, but they were so small that the dealer gave me two additional apples. I find that this made them cost just a penny a dozen less than the first price asked. How many apples did I get for my shilling?"

18. (*The Problem of the Dutchmen*)

"Three Dutchmen, named Hendrick, Elas, and Cornelius, and their wives, Gurtrün, Katrün, and Anna, purchased hogs. Each bought as many as he (or she) paid shillings for one. Each husband paid altogether three guineas more than his wife. Hendrick bought twenty-three more hogs than Katrün, and Elas eleven more than Gurtrün. What was the name of each man's wife?"

19. (*The Problem of the Two Trains*)

"Two trains, each containing sixteen cars and an engine, met as the diagram indicates. Each switch will hold just seven cars and an engine, or eight cars. The trains pass each other and move as shown in the diagram. How do they do it? How many reversals of each engine?"

20. (*The Angle Problem*)

a. "Determine how many degrees in an interior angle of an equilateral triangle;

b. of a square;

c. of a regular hexagon;
d. of an octagon [10a]
e. of a pentagon;
f. of a decagon;
g. of a regular sixteen-sided figure."

21. (*The Milkman Problem*)
"A dairyman in preparing his milk for public consumption employed a can marked B, containing milk, and a can marked A, containing water. From can A he poured enough to double the contents of can B. Then he poured from can B into can A enough to double the contents of the latter. Then he finally poured from can A into can B until their contents were exactly equal. After these operations he sent the can A to London. The problem is to discover what are the relative proportions of milk and water, that he provides for the Londoners' breakfast tables."

22. (*The Density Problem,—Fresh-Salt Water*)
"Does the water-line rise or fall as a boat passes from fresh to salt water?"

23. (*The Train Problem*)
"I asked a farmer how far he had to drive from the station, and he replied: 'If I get out at Appleford, I will have as far to drive as if I had gone to Bridgefield, fifteen miles farther on; and if I change at Appleford, and go thirteen miles to Carterton, I will still have the same distance to drive. Carterton is fourteen miles from Bridgefield.' How far did the farmer have to drive?"

24. (*The Density Problem,—Density of the Earth*)
"If the earth were of equal density throughout, it would be about two and a half times as dense as water; but it is about five and a half times as dense; therefore, the earth must be of unequal density. What is your reaction?"

[10a] The word 'regular' was omitted in the statement of the sections d, e, f, of this problem in order that the observer would be presented with an impossible task if he comprehended the statement of the problem. In this way an additional factor was presented. Several other problems contained similar situations. Any confusion which might arise therefrom was desirable because it gave opportunity for determining the readiness of the recall of this factor. (See p. 76.)

25. (*The Water Problem*)
"How much deeper is a body of water than it looks?"

26. (*The Vision Problem*)
"The *minimum visibile* is the least magnitude which can be seen; no part of it alone is visible, and yet all of its parts must affect the retina in order that it may be visible; therefore, every part of it must affect the retina without being visible." Is this true?

27. (*The Nephew Problem*)
"How can each of two men be uncle and nephew to the other?"

28. (*The War Problem*)
"The object of war is durable peace; therefore soldiers are the best peace-makers. Do you agree?"

29. (*The Cramming Problem*)
"Cramming is detrimental rather than otherwise; for I have noticed that, no matter what the subject is, I invariably write a poor paper when I 'cram' and a good one when I do not cram. What is your inference from his experience,—cramming is detrimental or not detrimental?"

30. (*The Proposition Problem*)
"Outline the kinds of propositions as used in logic."

31. (*The Aristotle Problem*)
"'Aristotle is living' and 'Aristotle is dead' are both intelligible propositions; they are both true or both false, because all intelligible propositions must be either true or false. Explain."

32. (*The Linnets Problem*)
"What fallacies are contained in the following? Linnets when shut up and educated with singing larks,—the skylark, woodlark, or titlark,—will adhere entirely to the songs of these larks instead of to the natural song of the linnets. We may infer, therefore, that birds learn to sing by imitation, and that their songs are no more innate than language is in man."

IV. Results

The chief purpose of our investigation has been to analyse and describe the mental processes involved in the act of recalling logically related materials,—problems, and the observers' solutions of them. We shall treat individually, in section *B*, the recall of a number of the problems, in the order given above. From their descriptions it is hoped that the reader will be able to see indications of general tendencies. These tendencies as indicated by the total number of single recalls (600) will then be treated under separate headings in section *C*. Since a description of the act of recall would be unintelligible to a reader who did not possess a knowledge of the observer's act of solving the problems, it will be necessary to preface our statements of the results of recall by a brief statement of the processes employed in the act of solving. In section *A*, then, we shall describe a few of the general features of the solving. But in order to make clear the statement of results of recall in sections *B* and *C* a number of the particular facts of solving will necessarily be treated there.

A. Solving of the Problems

For the solving of the earlier problems in our list, we presented the physical objects so that the observer might perceive and manipulate the parts to aid him in his thinking. The solution of these problems depended upon the perception of the relations existing between the parts of the object presented. In fact, all of our problems are similar in this respect; but those later in our list involve many relations which deal with human experience. They are considered to be more abstract and more difficult by most observers.

In not a single instance did any observer solve a problem without having recourse to some physical object, which he either perceived or imaged. It is interesting to note that most observers examine this object of perception with considerable care, even checking up incidental features before attempting to solve the problem. In the presentation of the 'dot problem' (see p. 38, problem 1), the number of circles employed are given both in the description and in the diagram. Nine of our eighteen observers first of all added the numbers in the groups and counted the circles in the diagram as a check upon the statement of the problem. Only after this superfluous preliminary step did they begin their attempts to solve it.

All but two of our eighteen observers made at least half a dozen definite trials which they wrote, and often many more trials in visual imagery that they did not write. At first they attempted to find a variation from the original which would constitute the solution; ten then attempted to write the possible combinations of the sixteen circles in groups of six, six, and four. No one continued this line of attack to its logical end, but returned to his first method, because to make all the possible combinations of six, six, and four became as tedious in itself as the work of moving two lines in every possible way. When they tried to match up these groupings of six, six, and four, with that presented, they found that they had employed different numbers of lines, and had never used the same number as there were in the original grouping as presented. Further, in some of these forms, not a single line was in the same position as in the original grouping. The observers perceived this extreme variance, and they also saw that to form such a grouping every one of the nine lines in the original must be moved; consequently they returned to the attempt to move two lines to the proper position. By a long process they eliminated many possible moves until they finally hit upon the correct ones. The two other observers perceived more relations and began their attempts at solving in a more rational manner.

It is not an easy matter to distinguish sharply between the use of trial images or assumptions, and the act of reasoning One might extend the meaning of the term 'reasoning' to include all trial solutions; but as we shall use it in this paper, it will embrace only those trials in which the observer foresees the necessity of the step, or in other words, when he sees the result of a step before he takes it, opposed to those trials in which he works without seeing the necessity for a single move. Only two of our observers, during the solving, used reasoning in this sense of the word. The steps of reasoning that these observers attended to were, first, the fact that the two circles in the center group must be allotted to another group in order to make groups of six, six, and four; second, the question as to which line of those bounding this group of two should be moved. Then they saw that if the lower boundary lines be left unmoved and the central dividing line below be removed, one group of six is determined; and they then concluded that one of the two lines on the upper boundary may best be removed. This leads directly to the solution. The following introspections are examples of reasoning in the first solving.

Ex. 1.[11] Obs. *Wh.* [The dot problem. 65 sec.] ("Introspect.") "My attention fastened upon the group of two, and I saw that in order to make groups of six, six, and four, one of the boundary lines of this group would have to be removed. I looked to see to what group these two should be added. Then I saw that by removing the lower line the two groups of three each would be connected and form a group of six, so the two circles should be thrown open above and be added to the group of eight. And this I imagined as done by viewing it as if just one of the lines were removed. . . ."

Ex. 2. Obs. *Ck.* [120 sec.] "I went at it pretty systematically to see how many of the heavy lines could be left unmoved. I saw that the top group had to be divided somehow. Then there came visually,—six below by removing only one line [the lower vertical line]. A top boundary line then had to be taken from the center group of two. I imaged each line taken. Now I had the center group broken and also the lower group of six determined; and I took just a moment to see where the two lines should be inserted in the large group of ten above. I saw the four at the left and placed the lines vertically in the center, finishing the problem. . . ."

A majority of observers did not at first perceive a relation that would bring them to a ready solution; instead of so doing they fastened their attention and directed their manipulations upon some unessential relation or false assumption. For instance, in the case of the 'block T,' the right angle was the first thing that the observers tried to fit or fill in. This kept them busy a long time and when they found that they could not accomplish it they concluded that the T could not be made. A new assumption could not be found and so a sheer trial and error method of fitting the pieces was begun with the hope that they would soon fall together in such a way as to look like a T. Only one observer perceived the relation and proceeded to work on the basis of this observation, that an edge of one of the pieces must necessarily fit upon or match an edge of another piece. This method brought the solution in a minute, whereas the other observers worked for ten minutes or, in some cases, almost an hour. This same difficulty of finding a new assumption or method of solving was present throughout all the problems. The essential feature in the solving of any problem is the attending to a particular portion or aspect of the physical object about which the problem is built. But ordinarily the observer fails to do this and some relation is overlooked,

[11] The numbers appended to the examples of introspection are inserted for future reference. The quotations within parentheses are the experimenter's words; the quotations not in parentheses are the introspective reports of the observer. Insertions by the experimenter for explanation are enclosed within brackets. The time in each case before the beginning of the observer's response is noted in seconds.

which must be seen if he wishes to reach the solution methodically and not accidentally.

B. The Recall of Individual Problems

Problem 1. (The 'Dot Problem.') It will be our purpose here to present particular facts of the recall of problem 1. These are: (1) the cues necessary to evoke a recall; (2) the time spent in arousing the first datum of recall; (3) the structure of this datum; (4) the nature of the first imagery; (5) aids employed in definitizing this imagery; (6) stability of the later definite images and the significance of this stability; (7) the dominant kind of imagery; (8) trial images and their aid to progress; and minor points of detail. This treatment, it is hoped, will present to the reader a definite view of the act of recalling.

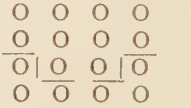
Although this problem had not been named when presented, and actually is not a dot problem, the observers were given the cue "Recall the dot problem." The name 'dot problem' was used here in order that the observer might be started on his recall without being given a single factor of the problem. In only four cases was this cue insufficient, whereupon the words "The circles problem" were added. Three observers recalled the problem in all its essential details, except for the fact that they reproduced dots instead of circles. One recalled this problem when asked to recall the 'box problem;' for her the groupings were considered as boxes. In one case, in addition to the cues 'dot problem' and 'circle problem' it was necessary to add another cue, which was 'the groupings.'

In no case was there any immediate memory that such a problem had been solved. The length of time between the request to recall and the first traces of recall was never shorter than several seconds; in certain instances it was about five minutes. In these slower reactions, the observer reported an initial feeling of familiarity and a stage of warming-up; then a vague, schematic, visual image. This image slowly became more definite and took on detailed characteristics, such as lines, circles, groups, etc. This progressive development of the process of recall is illustrated in the following introspections:

Ex. 3. Obs. *Fs.* ("Recall the dot problem.") [5 sec.] "There was almost immediately a rather slight tension. The focal things in consciousness were the sound of your voice which lingered in consciousness, and the vocal-motor innervation to say 'dot,' with questioning inflection. I actually whispered that word two or three times. Then

a single puzzle occurred to me, but I cannot now remember what it was. It was dismissed almost immediately as not having anything to do with dots. Visual image of that diagram of the train puzzle, that is, the paper with broken lines appeared; but that also disappeared almost instantaneously. The star puzzle appeared, connected with the word star, as a visual image. The puzzle was ten stars in lines, but I simply saw the ultimate solution. That also disappeared immediately. [This latter is not a problem of our investigation.] I did not recall the problem." ("Can you get it now?") [22 sec.] "Just a moment ago I had a flash of familiarity and nothing more than the vaguest sort of a visual image of dots, indefinitely located in space,—a faint recollection that I had had such a puzzle before. It was not present so clearly as that,—only a few dots arranged in a square, perhaps sixteen in all. The visual image was almost instantaneous, but of increasing clearness. . . . . I then started out with familiarity and these dots, together with a verbal motor, half-formed sentence, of which the general purport was: 'Those dots are to be divided by a certain number of lines into certain definite groups'; but I was too intent on going ahead with the problem to have that sentence come as completely as I have given it. Lines attracted my attention so that I did not seem to go on to work out some way in which this thing could be divided off. I immediately set down these dots in that square and found the tendency to divide them off in various ways with lines. Nothing systematic about that because I was unable to recall the way they actually had been divided off. A tendency to try and divide them into equal parts. I abandoned the effort and began to introspect. . . ."

Ex. 4. Obs. *Ck.* ("Did you ever have a problem which was concerned with dots?") [30 sec.] "No . . . (Or circles?") [55 sec.] As nearly as I can say, there was a relaxation of the diaphragm, and perhaps a change in breathing. Then there gradually came an uncertain visual image of small black circles arranged in a square, four or five each way. At first this image seemed to be as much an invention of my own as anything else. Then there came some vague organic reaction; then came a visual image of two or three straight lines dividing off these dots from others. The problem was then inferred to be: three lines (I am not certain of the number) given in one position; I am to move them so that the dots could be put in some sort of grouping, I do not know what, at all. I cannot decide whether there were four or five dots in a side, but each image was a square. I think the final grouping had something to do with six. This was a pure inference from the arrangement of the dots in my first visual image." [She called them dots, but now in her drawing to illustrate her visual image, she drew them correctly as circles.] "I now, while drawing, recalled vaguely (visual image) that the lines were one for each dot. This was at first very indefinite. The dots going around a square corner were counted in one group. This is the lower six. The dots in this arrangement now came out clearly, simultaneously.



Ex. 5. Obs. *M.* ("Recall the problem that you solved after the counter one.") [40 sec.] "I had a series of visual images, in the following order; the discs, the railroad, the capital T, and the circles.

These were located in a visual schema, rather than detailed images. Then at last the circles in my visual image cleared up. At first they were dim—perhaps three rows of four each. When I started to set them down they cleared up nicely into four rows of four each. The heavy lines began coming in. . . ."

Ex. 6. Obs. *Bk.* ("Recall the dot problem.") [35 sec.] "I imaged the typewritten sheet with the diagram on it. Can almost read the statement of the problem. A '6' and an '8' come in. My visual images are so dim that I do not know whether they are constructed or recalled. I am not sure that there were four dots in each line and each column." [Draws.] "Oh, the heavy lines come in, and they make the whole figure lopsided; also a lower line keeps coming in. These images are multitudinous and as involuntary as reversible visual illusions. . . ." [The lopsided figure which came in at this point was in reality a visual image of the correct solution, as was clearly shown when Bk drew a picture of this visual image.]

Ex. 7. Obs. *Wh.* ("Recall the dot problem.") "Immediately I got a vague image of the original paper with dots on it,—with no definite arrangement, however. The first image was more like dots than ciphers. Now I see ciphers. The figure that I first saw was in the shape of a cross, then it became a square. [Pauses 2 seconds moving lips as if counting.] I cannot tell you the number; I will draw it." [While drawing he said:—] "There were 12, I think." The fact that there were sixteen circles did not occur to my consciousness until they had been drawn. Just after I had drawn them I had a visual image of the lines within the figure. . . ."

The number of circles was never definitely recalled at first. The observers often began to record dots and then recalled that dots had never been seen, but circles instead. Also the number of circles was definitized in writing them, as is shown in introspections 5, 6 and 7. Usually the visual image preceded the writing; but it was very often verified or corrected by the perception of the trial forms that the observers wrote on the page.

The above examples of procedure in recall indicate also the nature of the imagery in regard to its instability and its development. We see a growth in clearness and detail, also a correlation between clearness in details of the image and recall of the steps in the solution. This is more or less typical of all eighteen observers. In Ex. 12, p. 53, the haziness of some of the lines signified that these had been the ones shifted in the course of solving. At times the images were stable and slow in development; but more frequently, particularly in the cases of *Wh, Ck, Fs, Bl,* and *Bk* they were multitudinous and very unstable, as in Ex. 6, given above, and in Ex. 8.

Ex. 8. Obs. *Bl.* "I have recalled the general form of the problem in indefinite imagery. Can see fluctuations of groupings but cannot say what they are. Am sure that the problem is made up of circles,

divided into groups by heavy lines, but cannot see how many of either, or their exact position, because my imagery is too vague and fleeting. . . ."

In almost every case the images that appeared at a time when the observer was attempting to recall were very significant to him. Aided by these, he recalled or constructed others which formed a problem,—often the correct one. The following introspections are examples of this.

Ex. 9. Obs. *C*. [60 sec.] "Just a vague visual image of the lines. The problem is to move two short lines so as to divide the spaces into 2, 6, 1, 4 instead of . . ., Wait just a moment. I am about to get it now." [He draws 1 min.] "All that I remember clearly is that I moved two lines; I see myself moving them. However, I am not very sure just where these two moves were made." [Pauses 5 sec.] "I have a vague visual image of a written 4, 6, 8; no, it came more in vocal motor imagery. It seemed to come spontaneously with the problem itself. The moved lines came in quite clearly." [Here he drew the position of the two lines when the problem is solved.]

Ex. 10. Obs. *W*. ("Recall the dot problem.") [13 sec.] "At first no more than the word 'dots' shifting in and out, auditory imagery; and then black dots faintly coming and going in visual imagery. Just now there comes a visual image of dots arranged in a sort of cross, and vertical lines appear between two of them. This brings the certainty that the problem is to group dots, the certainty being merely the fact that it is a fixed (stable) image with fixed meaning. . . ."

Certainty in recall is apparently closely related to stability of images, as is illustrated above, in Ex. 10; but it is not safe to generalize from recalls of a single problem, by only eighteen observers. Later (p. 72) we will discuss this point, and also the factor of meaning as these are found occurring throughout the various problems.

The visual was the dominant mode of imagery, in the recall of this problem; this was to have been expected, because little use had been made of other kinds of imagery during the solving. In addition to the visual imagery in the solving however, every observer employed words, and movements of the hand in drawing the trial positions of the lines; but the solving was carried on almost exclusively in visual imagery. In the recall the only imagery besides the visual was the hand motor, or vague kinaesthetic, which was bound up closely with the visual. Only three of eighteen observers noticed this kinaesthetic imagery.

The trial groupings that had been made during the solving (p. 46), never came clearly to consciousness again during the act of recall. In half of the cases they were recalled as vague visual images, so dim that not a single line or group was seen distinctly. In a few cases, the remembrance that

many unsuccessful trials had been made in solving was merely a vague memory of the time spent in the room trying to solve the problem upon the first day. The general situation in the room was most definitely remembered, together with the visual image of the solution and the fact that the problem had been solved.

The observer who, in the act of recalling, has an abundance of trial images is the one who makes the most rapid progress. But these are never the same images that occurred during the solving. It is not surprising that this proved to be the case because not only did his present goal lie in a different direction but he also set out from a different starting-point. Only one of the hundreds of trial images that had been employed in the solving was recalled, and this incidentally, after the essentials of the problem,—the form of the final solution, the presented form and the complete manipulation which was appropriate for the act of solving,—had been recalled. Not even the steps in reasoning which appeared during the solving were reinstated, as will be shown below.

Neither of the two observers who used the steps of reasoning in the solving (p. 46) recalled them, but recalled first the visual image of the solution and then the image of the form originally presented, with the attention directed to the two lines and their change of position from the original to the final form. Hence they recalled what they had seen and done; they did not recall the reasons why these particular moves should be made.

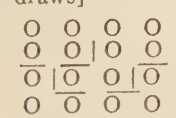
First recall showing the failure to recall the steps of reasoning given in Ex. 1.

Ex. 11. Obs. *Wh.* [Continuation of Ex. 7.]

[His drawing]

"This is the original presentation, and the problem was to make the groups six, six, and four instead of eight, six, and two. I had a visual image of the typewritten figures 6, 6, and 4. I think my instructions were: Change any two lines. This comes to me now in the visual images of seeing myself move them." [Pauses 90 sec.] "I think I have an image of the finished figure."

[He draws]



[Pauses 30 sec.] "What troubles me is to get the lines there by two moves." [Studies 150 sec.] "I have it,—another line below in the original visual form. This was slow to come in, although I attended strenuously to the first drawing above, trying to make it clearer in visual imagery, and to see another line in it, which I might place vertically above in the center of the final drawing."

First recall, showing the failure to recall steps of reasoning given in Ex. 2.

Ex. 12. Obs. *Ck*. [Continuation of Ex. 4.] "In this visual image there were one or two lighter lines fleeting down below somewhere. To interpret, the fact that they were lighter seems to indicate that they do not belong there permanently. The dim gray lines tended to move into the black ones and stay there. This seems to represent the problem and the solution." [Pauses 84 sec.] "My first thought was to move these two lines up here (copying my visual image from above). My act of solving consisted in moving the lower line visually, and that leaves six immediately. Another scrap of memory is that the two lines, after being moved, stand end to end. This came in first as a visual image." [Pause 35 sec.] "Can it be possible to divide the circles into four groups of four each? I have been trying to imagine lines to do it, and to see if they looked natural that way. I did this in three or four ways very quickly. All I am clear about is that one or both of the two center dots is to be counted with some others,—a remnant of the first definite visual image and an imaging of how I can make it conform. I am surest of the general longness of the group. I cannot get any other familiarity clue. Each construction looks as unfamiliar as any other, so I do not know which, if any, is correct. I could reconstruct the problem by making groups of six, six, and four, and taking the smallest number of moves necessary to make this number. From the memory image of the two vertical lines in the center above as the final solution, and the lower group of six, I get the original form; and I assume (visual image) that I got one of the vertical lines from below. That makes it six, six, and four. . . ."

We may call this last example an act of reasoning from a basis of two definite memory images to the formulation and solution of the problem. But these steps are not the ones which she took during the first solution (Ex. 2, p. 47). Several other clear cases of reasoning occurred only in the recall of this problem, and these consisted in filling in similar gaps between the bits of definite recall.

Recognition often resulted when the gaps were properly filled, as the following example shows.

Ex. 13. Obs. *M*. [After drawing the final and the original forms as he had constructed them by filling in the gaps between his memory images, the following introspection was given.] "The two circles were shut off in the center, I remember visually. I think that we were to change two lines so that six circles were inclosed in every pen. The most clear is the group which I draw here. I read out of my diagram that there were groups of eight, six, and two at the beginning. The first six is remembered in the form of the visual image of the group. The clearest part of my remembrance of the original presentation of the problem is the diagram, and it is not yet definite nor certain. The 'two moves' was auditory as if I heard you reading it.[12]

[12] "I have not now been able to recall the words visually." [It is interesting to note that the statement of this problem had been read to the observer for the first solving.]

"After I had completed the figure I tried to prove it to see if two moves were necessary. It does not work. I see I have eight lines in the first, and nine in the last drawing. I have not thought which lines of the original figure were moved to make the final form. The figures were separate visual recalls and not related to each other as they would be if the one were formed from the other by the shifting of two lines. Now I have the impulse to insert a line into the center group between the two circles. Oh, no. There it is vertically at the bottom" [dividing the group of six into two groups of three each]. "This is very convincing, and is verified by the image I now clearly recall of it. . . ."

The above examples show that the recall of this problem has been a keenly active process, and that there was much searching and striving to recover the problem and its solution. As was stated earlier, the observers employed much imagery in recall that was of a constructive sort, in order to see if any familiarity would attach to these created images that might aid in recall. Many observers found this procedure very profitable, and every one used it to some extent.

Every observer busied himself unintentionally first of all with the recall of the solution; and in every case, the first definite thing to appear was the visual image of the drawing, or a part of it, that they had made in their solution. The visual image alone constituted the structure of this recall. Rather surprisingly, this visual image, in almost every case, was recognized as the solution, from its being seen in one's own handwriting, on the page at the end of a number of vague drawings (unsuccessful attempts at solution), sometimes from seeing one's self leave the room, or just getting the solution as the time was up, from its familiar toning, from one's feeling of satisfaction, or from a relaxation of attention that occurred with the appearance of this image of the solution. When this point had definitely been settled, the procedure tended toward the recall of the original figure. Seldom was this attained by pure recall, but most often by a process of constructing from the recalled images. Every observer proceeded in a direction opposite to that which he had followed in solving, the recall terminating at the point where the solving had taken its start.

Certain individual variations have been mentioned in the discussion of this problem but they are not of sufficient importance to merit further treatment here.

2. The Block T, and the Ring Problems. (See p. 39, for the statement of these two problems, both of which are mechanical puzzles; and see p. 47, for the statement of the method of solving.)

After an interval of a month or so, the recall showed the following main points. An assembled T, approximately the size of the original was clearly recalled, then the lines uniting its parts appeared slowly,—all in visual imagery. These images did not become clear however until after the attention had fastened upon the right-angle at the juncture of the stem and the crosspiece. This angle was recalled as the jog in the end of the large piece, whose location in the T had been difficult to discover, but, when once found, made the assembling a very simple and easy matter. The lines were then seen very clearly in the visual image. The word "bias," which carried the meaning of the crux of the problem for one observer, was recalled first (visual image) by him. Another recalled the diagonal lines, which meant for him that they were the essentials of the problem. Never were the pieces severally recalled, in definite form, previous to the appearance of the image of the assembled T. Five of the eighteen observers, who had recalled the essential features of the problem, were unable to recall or to re-construct the pieces of the T. Nearly all did more or less constructing by many trials to complete a definite recall of the forms of the pieces. This mode of procedure is well illustrated by the following example.

Ex. 14. Obs. *Bk.* ("Recall the first problem you solved.") [10 sec.] "I just saw a visual image of the constructed T, at first. Then I saw two straight lines running from north-east to south-west, and diagonally across the T. I have a distinct visual memory that the notch is in the center piece. On second thought this seems to be incorrect; I do not quite remember the right shape. I have nothing to verify it by. I feel pretty sure that in general it is correct. My process of doubting the correctness of this center piece consists simply in looking at it and in feeling an unsettledness and uncertainty. I try to call up the image of the original, but it does not come. I remember (motor imagery) trying to get them to fit together. I also have visual images of gray pieces." [He drew the constructed T, looked at it, and changed it, making it wider in the stem.]

The main point of interest in the 'ring problem' consists in the fact that every observer first recalled the principal characteristics of the ring and the disc, and then the twist that released the ring from the center hole, all in visual imagery. One might suppose that the twist, the motor act which brought the solution, would be recalled by means of a motor image; but in the case of half of our observers the motor image did not appear in recall; and in the case of those observers for whom it did appear, it did not make its appearance until after the visual image of the act had become

clearly established. No tactual images were aroused although, like the following problem, it appealed to the tactual sense.

Though all the observers had worked with the holes that are half way through the disc, only two now remembered that they existed; and this only after all the other characteristics had been recalled. The effort to take the ring off the edge, although it involved a definite motor sensation, was invariably remembered in the form of a visual image of the act of twisting the ring and the feeling of disappointment at the failure. The situation, the room and the surroundings were most clearly recalled.

3. Problem 4. The Touch Problem. (See p. 39 for the statement of this problem.)

In the recall, the first definite remembrance was a visual image of the sheet of paper held up against the forehead; then there appeared tactual images of the earlier experience during the solving. This tactual imagery was so vague that it was not employed in definitizing and furthering the recall of the problem nor had it been so employed in the earlier experience. Visual images of the dots and lines as the observer had drawn them on his paper during the experience came clearly to consciousness immediately afterwards. Then the crux of the problem,—the existence of the two points of view,—was recalled in visual imagery. Note the two illustrations below,—Exs. 15 and 16.

The accuracy of the perception of the direction and distance of one dot from another by the tactual sense proved to be far inferior to that of the visual sense; and after five minutes those tactual impressions which had not been transposed into visual imagery and not written down were incapable of being revived.

Ex. 15. Obs. *Wh.* ("Recall the problem of touch.") [60 sec.] "I had begun to visualize even before you got through speaking, and my attention was concentrated upon this alone.[13] Then I repeated 'touch' again; strain in my forehead, and immediately had a visual image of the paper on my forehead, and tactual imagery of the day's experience. I knew it was the problem. Definite reconstruction of the presentation,—pressure images akin to your first day's presentation

[13] I had a definite set in consciousness to act quickly. This consisted partly in auditory images of your voice, and in repeating the word 'touch' to myself, hearing your words again, seeing myself standing over an iron puzzle of yours [number 4] then the actual tactual perception of the counters of your 'lozenge' problem [number 8]. That is a touch problem, but not the problem you want." [The above illustrates the series of irrelevant images that often precede the recall. The remainder of the introspection exemplifies the points cited in the discussion.]

came. Then I visualized the drawings I had made, and saw myself making them.[14] . . . I saw an image of the inverted Z that I had drawn because I had not perceived the direction in the first experiment. Experienced again the complex situation. I imagined myself drawing in both directions,—to the left, and to the right. Only now when I have a visual image of myself putting the paper up to my forehead do I visualize the stimulus as it would appear from your point of view. I could not recall it without imaging the whole situation."

Ex. 16. Obs. *G.* ("Recall the touch problem") [11 sec.]. "Nothing in consciousness at first except the auditory image of the word 'touch.' . . . Then a visual image of you putting a paper on my forehead, a flash, no readiness, no leading up to it. A visual image of my forehead. Then you put lines in a certain order, and had me draw lines that corresponded to those you had drawn,—vague, visual images of what I drew."

4. Problems 5 and 6. Memorization. (For the statement of these problems see pp. 39 and 40.)

Problems 5 and 6 were presented with a view to bring out individual differences between the various observers in that the memorizing of the numbers would here permit the more abundant use of vocal motor imagery than others of our problems; thus the memory of an individual who is inclined to use the vocal motor in preference to other kinds of imagery, could be compared with his memory for those problems which do not admit of so ready a use of vocal motor imagery. The recalls showed that no one made exclusive use of vocal motor imagery in his revival of the problem; nor were these problems remembered more permanently than any others.

A secondary interest in these two problems has been to see whether the perception of relation would be recalled apart from the numbers between which the relation obtains. Each problem was recalled three times,—at the end of one minute of memorizing, one half hour later, and again one month after memorizing. (During the half-hour interval the observer was strenuously employed throughout the thirty minutes in solving other non-related problems.) In not a

[14] Attention turned back to forehead. Auditory image of your voice saying 'reproduce.' Saw you taking paper away, not letting me see what you had written on your paper against my forehead. The actual images are not plain. The dots, etc., are indefinite." [Pauses 35 sec.] "I have a visual image of the crescent, the Z, the triangle, etc., that you drew on my forehead as I drew them on the paper. I had an auditory image of the noise and of the punching of the pen through the paper." [Pauses 31 sec.] "Now there came and surprised me, a mass of imagery rich in meaning, which aroused much pleasantness. I was amused when I saw through it, and reflected that I could not get it right the moment before.

single instance during the first two recalls did the remembrance of the relations come to consciousness before the remembrance of the numbers themselves. In other words, the terms were remembered as unrelated data at a stage before the relations came to consciousness. However, the successive appearance of two numbers that had been consciously related was a common occurrence. For instance, in problem 6 the solution involved a consciousness of the fact that 10 is twice as great as 5. In the recall of this problem after half an hour, three consecutive stages were clearly differentiated: (1) a consciousness of a concrete, unrelated 5, succeeded almost immediately by (2) a consciousness of a concrete unrelated 10; (3) and not until the third stage was the observer aware of the relation, namely the fact that 10 is twice 5; and in certain instances the third stage was wholly absent. But in recalling after an interval of a month, the revival of the fact that certain relations had been perceived was first to appear after the visual schema, and while attending to this fact the observer recalled the numbers. However, unrelated numbers were also correctly recalled in numerous instances. The following group of typical examples illustrate, first, the introspective report of the perception; second, the report of the first recall after half an hour; and third, the report of the recall after one month.

Ex. 17. Obs. *Fs.* [Introspection upon the act of memorizing problem 6.] "When I picked up the diagram I looked first at 12, then at 27, and found that 27 was not twice 12. I abandoned that and glanced around in search of a number which would be twice 12. An '*Aufgabe*' to hurry, present only in the tendency to look at certain figures quickly. I chose 7 and glanced through to see if there was a number that was twice 7, but I did not get through the whole list. I selected another hurriedly. Not a systematic finishing out of the procedure which as far as it went was somewhat systematic. Not aware of having known definitely what I was looking for, *i. e.*, I did not say, '7' and '14,' and look for 14, etc. I said '5' and then glanced through. My glance happened to fall on 10, and I said, 'Two times five are ten.' The procedure adopted then consisted in glancing through to find numbers that were multiples of three. I hit upon 18, then upon 21 with 7, and finally upon 48. I glanced across at 12, having remembered that it was there. It is interesting to note that I glanced across instead of imaging 12. No other numbers contained three so I repeated the others. . . ."

Ex. 18. Obs. *Fs.* ("Recall what you did half an hour ago.")[15] "First thing was a visual spatial schema localized down here, representing the general outline of the problem. Immediately attention

[15] This, of course, refers to the problem of memorizing. The experimenter deliberately presented his instructions in this vague form in order that the clue to recall should be as indefinite as possible.

seemed to turn to this side of the schema; I can not now say whether I visualized the figure and schema simultaneously, or the figure first. I wrote 27 on my paper, but there was a period of deliberation as to whether 27 or 12 belonged there, both being present in vocal-motor terms, as though I were speaking the words 'twenty-seven,' and 'twelve' with an inflection of questioning. A definitely localized 19 occurred to me in visual terms, accompanied by vocal-motor; and without hesitation I then wrote 27 and 12 in this form. Then 'three' came and remained as vocal-motor setting in consciousness; a slight tension which did not relax until I had written all the three, 27, 12, and 19. When I set 18 down below, it seemed as though it might belong over above. I thought they belonged approximately in the same line. That is present to consciousness simply as an act of setting it down. The next that came were the 16 and the 5. Then came 8 and 7, and I hesitatingly set them down with question marks. Then came the 10 localized down below. . . ." [The 5 and the 10 were recalled separately and their relation was not recalled.]

Ex. 19. Obs. *Fs.* [Second recall, after an interval of one month.] ("Recall the problem of memorizing.") [43 sec.] "At first there was a period of blankness; I was aware of nothing but you and the words you had just spoken, the latter in auditory imagery. Then there came a visual image of a sheet of paper. My attention was primarily directed to the numbers written on it in a vague elliptical shape. I remembered my general method of procedure: reading the numbers hastily without any plan, then catching myself up in the middle and looking for associations, looking at different numbers, bearing one in mind and glancing over the rest to see if any relations could be found. This remembrance consists in visual imagery of glancing at numbers. The auditory imagery (own voice) of the word 'divisible' appeared just now, which meant that I was looking for one number that was exactly divisible by another. Then 18 came; it was visualized down there. Aware that '18' is a multiple of 6,—'6 times 3' (vocal-motor images with dwelling on 6, which I interpret as an awareness that I used 6, in some way connected with 18). [The 18 is correct, but the attaching of the multiple 'three' to 6 to form 18 is incorrect. Note that after the 7 was recalled below, the idea of the multiple three was employed, which finally led to the recall of 21. The 19 came to consciousness unrelated.] Visual image of 7; then said to myself '6 times 3, 6 times 7, 6 times 7, 7 times 3,' which latter gives 21, and this I accepted as correct. A visual image of 19 appeared, then . . ."

From the above examples we see the same general principles that have appeared in all the problems so far treated even though this is a problem in memorization. The visual image is the first definite factor in recall, and is used very freely throughout the process. The recall progresses, by means of a multitude of trial images, from the vague visual image noted above through greater definiteness and detail until the whole problem is reinstated.

5. Problem 7. The Addition Problem. (See page 40 for the statement of this problem.)

For ease of orientation this problem may be introduced

by referring back to problem 1. Here also the presentation consisted, in part, in a visual structure; but in the former instance, the general form was that of a square, in the latter, an equilateral triangle. Although in this latter problem, the fact that the numbers appeared in a triangular form had no particular significance, since any other arrangement would have served equally well, yet every observer reproduced a visual schema, in the initial act of his recall. This schema, for all but two observers, consisted of a visual image of vague numbers arranged correctly in the form of an equilateral triangle. This image was clearly recognized as the form which had been observed in the solving; and although the problem could not now be formulated, yet this vague, schematic, visual image signified to every observer that he was in the act of recalling the addition problem.

The two observers, who failed to recall this triangular form of the group of numbers, recalled it as a square which served as the basis of their further recall, and which fulfilled for them in every way the same function as did the triangular form for the other observers.

This problem was more difficult to recall than those just described. It will be noted that the experimenter's cue contained no suggestion of the visual structure of the presentation; and if we may infer that it is necessary first to regain this form (from the fact that every observer recalled the visual schema before proceeding further with the recall), we may offer the explanation that this relatively retarded reproduction was due to the fact that our cue was only slightly suggestive. Several observers were not able to proceed from the first cue,—'the addition problem;' in such cases, after several minutes of non-productive search we gave a second cue: "You were to select certain of the numbers which when added gave the sum required."

In the case of two observers, a third cue was necessary: 'A sum was to be found from a group of numbers.' This aroused a schematic visual image of the form of the group of figures, and a recognition of the fact that at last a recall had been begun. It was one of these observers who recalled the group of figures as being in the form of a square. Several illustrations showing the general trend of recall are here given.

Ex. 20. Obs. *Fr.* ("Recall the addition problem.") [50 sec.] "First there came a visual image of a triangular group of numbers, some of which contained two digits, and others one; no part of the visual image was clear. Then 'Find three that make the sum fifty' came in vocal-motor imagery. . . ."

Ex. 21. Obs. *Y.* ("Recall the addition problem.") [64 sec.] "I cannot recall a thing. I believe that I never had that problem." ("You were to pick out certain numbers which, when added, gave you the sum required.") [The observer repeated these words twice and then said:] "I had a vague, triangular, visual schema; a group of numbers came, but I could not read any of them. Then I recalled myself sitting here selecting three of them or trying to select the proper three. This was mostly visual, too vague to be sure about the three. Now it comes again more clearly, the numbers '10,' '12,' another '5.' Tried many times to make a sum of 25 from three numbers; I got it and went out glad. I see myself leaving the room as if it were only half an hour ago. . . ."

Ex. 22. Obs. *G.* [41 sec.][16] . . . "Now I have a visual image of a triangle of numbers. I now see also faintly the words 'Three numbers that make 25'; also some vague motor imagery of picking out three numbers and of running my pencil around over the group. I took one number and added two others to it; I see myself as I did this. . . ."

In the last two examples it is to be noted that the number recalled as the sum to be found was 25 instead of the correct number, 50. This same substitution occurred in the case of three other observers. It may have been due to the fact that in the search for the sum 50, 25 was the first to be tried as one of the numbers that make the sum. A 15 was also seen and then a 10 was looked for, to complete the total sum of 50. No 10 was to be found in the group, so both the 25 and the 15 were abandoned. When at last the 19 and the 6 were selected and added, giving a sum of 25, the problem was practically solved, for the observers remembered visually that the group contained a 25 at the left; they did not need to turn the eye to it, nor to add it in to form the sum 50. That the 25 was there was sufficient for these four observers; and 25, instead of 50, had become the goal. The fact that the number 25 was ideated both at an earlier and at a later stage as the sum of the 19 and the 6, proved to be so impressive that, in the recall, 25, instead of 50 was taken to be the goal of the problem and the sum to be found.

Three other observers pursued the same general route; but they again found the 25 and added the three numbers, making 50, the sum required. It is to be noted, in contrast to the recall by the four observers who did not complete the sum 50, that these three recalled the 50. Two observers found the 25 and the 6,—which gave a total of 31,—and then

[16] "At first I was conscious only of your words, 'addition problem' in auditory imagery; then a visual image of 152, 3, then a column of figures, but these were imagined, not remembered. I remembered that the sum of certain numbers should be 40, but was aware that this was another problem."

found the 19; but they did not add the three numbers after the 19 was found. Consequently, for these two observers the first number to appear in recall, two months later, was a visual image of 19. From this they proceeded to the recall of the visual image of a typewritten 50, as presented to them in the original statement of the problem. One observer, who carelessly found the correct three numbers that made a sum of 50 by simply choosing the numbers and not adding them to see whether they made the required sum, failed to recall a single factor in the problem except the triangular shape. He recalled the numbers 100 and 55 of another problem as belonging to this one. When told he was wrong, he immediately recalled 25 as one of the numbers employed to form the sum (in definite visual and vocal motor imagery).

From these facts we may conclude that the goal or final step taken in the solving is most likely to be recalled first, and that this may then be transposed to stand for the statement of the problem.

To continue our comparison of the recall of this problem with that of the 'dot problem,' we may state that not only did the general form (visual schema) appear first and seem to be most important because it constituted the starting-point of both problems, but also that many trial images which had been employed during the first solving of both were never recalled. The first trial image which had been employed during the original sitting in the search for the solution was frequently recalled, but practically no other members of the original series. The contents that were most subject to recall were the visual form of the group, the successful trials, and the relative spatial position of the numbers employed in these trials.

For seven of our ten observers, the relative positions of the three numbers in the visual schema were more definite in recall than were the numbers themselves. Even before the numbers appeared, the observer clearly recalled the fact that the largest of the three had been the last used in forming the sum. When the numbers cleared up they were all in their proper places. The act of recalling them was very similar to a difficult visual observation where a searching or a peering into a dimly lighted region brings indistinct objects more clearly into view.

In their search for the proper numbers, several observers began systematically with the top figure of the schema and combined every possible pair of numbers with it to see if they yielded the desired sum. Then the number next to it

was treated with every other two, and so on, until the solution was reached. It was found that all observers remembered this procedure very clearly, in terms of motor or visual imagery; but the numbers themselves were not recalled until later. One observer, instead of searching for three numbers whose sum is 50, adopted the more rapid method of finding three numbers the sum of whose right-hand digits is a multiple of 10. This method yielded four combinations of numbers whose sums ended in zeros; but each of the first three combinations gave a sum of 60. The fact that this method had been employed was remembered in the subsequent recall, but with certain errors of detail,—a 5 was substituted for the zero at the right.

One might, of course, expect that in arousing a series of trial images in recall a process of reasoning would be employed. As for example, when one number is relatively stable in imagery, the other two which are brought up as trials shall complete the sum 50. This is shown very clearly in the following examples.

Ex. 23. Obs. *F.* [18 sec.] "At the outset I was conscious of nothing but the pyramid which was present as a vague visual image. Then came 19, and I immediately saw that 31 must be added to it to make 50." . . . "It seems that I was to find *three* numbers whose sum is 50. I recall the act of searching for three numbers in the group,—a partly motor and partly visual consciousness."

Ex. 24. Obs. *W.* ". . . . I had a 25 and a 10, in auditory, vocal-motor, and visual imagery, then a vocal-motor 15; 9 was localized visually near the bottom and to the left. Here I supplied a 6, in auditory imagery, and tried to localize it visually above and to the left of the 9.[17] ["The 9, in fact, was a part of the 19 of the number employed in forming the sum during the solving. The 6 visualized above and to the left had also been employed in the original solving. In an effort to form the 25 as the sum to be found, the observer in recall employed the trial images 8 and 7 but they did not persist. Later on the observer found the correct three numbers that form the sum.]

We find that all ten observers reproduced the visual form of the arrangement of the numbers, which grew clearer with the lapse of time during the act of recall. For five observers the position of the three chosen numbers then appeared, and later, some of the other numbers, especially the three that made a total of 50. Four observers recalled the appropriate

[17] These come again and again but with only a moderate degree of certainty; 8 and 7 come in but they do not persist and I am much less certain about them. I was in doubt as to whether 25 or 50 was to be the sum. This, I believe, was due to the fact that visual images of both of these numbers appeared, and attention shifted from one to the other, several times; this has left me in a state of doubt. . . ."

numbers correctly,—25, 19, 6; and four recalled the 3 which had occupied a conspicuous position at the apex of the triangular group of numbers. Three others spontaneously reproduced the 19 without recognizing that it was being recalled; two reproduced the 6, and two the 25 in a similar manner. Every observer when urged by the experimenter to guess or create if necessary in order to complete the formulation of the problem (which by the way demands a consciousness of the solution), correctly reproduced the three numbers which in the original formulation make the required sum; but they also reproduced more than half of the other numbers in the group. These facts are interesting in that they exemplify so clearly the predisposition, of the individual to follow a beaten path although he is confident meanwhile that he has struck out upon a new path, *i.e.*, he feels that he is 'guessing' and creating a wholly novel product. In many instances, however, certain of these imagined numbers seem to become endowed with a slight feeling of familiarity, and they then seem to the observer to be correct remembrances.

In the recall of this problem, the trial procedure proved to be the most efficient method. The observer who evokes many visual, vocal motor, or auditory images, and who then proceeds to test them is most successful in attaining a complete and accurate recall.

6. Problem 10. The Weight Problem. (For a statement of this problem see page 41.)

This problem, like problems 1, 4 and 7, will be treated in detail. Those lying near it in the series are so similar to it that they need not be discussed separately. For example, problem 8 bears a close resemblance to this one, both in its complexity of statement and in the number of short-cuts that may be employed. Problems 9, 11, 12, 13, 14, and 15 are all concerned with concrete materials. Since they draw upon various kinds of experience one would expect that certain observers would react more promptly to one problem than to another, inasmuch as their past experiences differ. However the following description of procedure with the weight problem is typical of more than a dozen of our problems.

In order to show the detailed nature of the recall we introduce here a few facts of the first solving,—the facts that are to be recalled. Seven observers failed to perceive that this problem really hinges upon the question as to what sort of weighing apparatus was to be employed. They simply proceeded to divide and to sub-divide the weight,—assuming that

an ordinary balance was to be employed. The other two observers promptly responded that to balance any number of pounds from 1 to 40 would require an infinite number of weights. This is correct, unless we consider that " any number of pounds from 1 to 40 " refers to integral numbers alone. These two observers were instructed to restrict their interpretation of the problem to whole numbers of pounds, and to proceed with the solving on that basis. In their subsequent recall they both failed to remember that they had had any difficulty with this point.

Every observer reported as his first solution, forty one-pound pieces. In only one instance, however, did this tentative solution appear in any way in the recall; and for this observer it was simply a bit of relatively non-significant imagery in the course of his interpretation. Five observers saw the necessity of adopting a certain kind of scale for weighing, in that each had a visual image of a balance, with its weights; and it seemed to them to be the only possible means of solving the problem.[18] These images of the weights for the balance were of appropriate sizes,—each one of a lower denomination being just half the size of that of the denomination next above it, except in the case of the three weights below five grams which are two, two and one.[19] This envisagement led these five observers to consciously imitate

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[18] No one reasoned far enough to see that one weight is sufficient to conform to the conditions of the problem. That is, if we conceive a balance-arm on which this weight may be shifted so that it is at any distance from the fulcrum, any weight up to forty pounds can be weighed with the undivided forty-pound weight.

[19] A frequent error in solving was that instead of correctly dividing the forty pound weight, first into two twenties, then one of the twenties into two tens, then one ten into two fives, and finally one five into two twos and a one-pound weight, the observer divided the last five-pound weight into a three, a two and a one. This error occurred in the case of seven of our ten observers, each of whom corrected it, however, during the solving. It reappeared for five observers, two of whom recalled that this had been an error in the act of solving, while the other three traveled the old route, made the error, then corrected it, but failed to recall that they had made this error before. We might infer from this result that the probability of the recurrence of this error was very great as compared with other errors. Or we might with greater assurance infer that the appearance of this error in the recall in many of our cases was due to the fact that the reappearance possessed rather the character of re-solving than of pure recall. The lighter weights could be either two ones and a three-pound, or one one-pound and two two-pounds. Nine observers employed the latter division; only one used the former, in his solution of the problem. Every observer in his recall revived the grouping he had formerly employed.

this method of division, and it possibly influenced the other three, who did not have the envisagement clearly in mind, to use this method. These envisagements were in every case vividly recalled.

Five observers instead of dividing the weight into halves, quarters, eighths, etc., began to build up the numbers from the one-pound weight that was first seen to be necessary, then to adopt a two-pound (which, combined with the one-pound weight, would weigh three pounds); then to choose a four-pound weight (which, combined with the one- and the two-pound weights, would weigh any integral number of pounds up to eight pounds), and then to choose an eight-pound weight, etc. They did not at first attend to the fact that the sum of their weights must equal forty pounds since that is the weight which is to be divided; but they finally corrected the error. These five observers recalled their method in the visual images of the figures which they had written on the paper, such as 1, 2, 4, 8, etc., as is shown by examples 25, and 28, pp. 67 and 68.

In the procedure of the five observers just described the process of adding weights of one or two pounds to obtain weights of greater magnitude was carried on until ten was reached. Then they saw that since by means of the one-, two- and four-pound weights, any number of pounds could be weighed from one to seven inclusive, and since with the eight-pound weight any number of pounds up to eight plus seven could be weighed, they need not count out the particular weights that are necessary to form each integral number of pounds up to the point where all the weights are placed on the balance and the sum fifteen is found; but they saw at a glance that all up to sixteen pounds could be weighed, whereupon they chose sixteen pounds as the next heavier weight. Three of the five observers who generalized in this manner, recalled more or less accurately the group of numbers that they had found to be the weights desired; and these visual images signified to them that they had begun with the one-pound and had tried each number of pounds, but in no case were these trials recalled. We should here emphasize the fact that visual imagery constituted the structure of a recall which served as a generalization. This has not been an uncommon occurrence in our results with other problems, as was pointed out earlier (p. 52). The few isolated images often signify a complete course of mental processes that had taken place during the solving. For illustrations of this, see examples 4, 11, and 12, also 25 given below.

Ex. 25. Obs. *B*. [Recall of the weight problem.] ". . . I had a visual image of my writing on paper, my former act of solving the problem. Then I saw a column of numbers,—1, 2, 3, 4, 5, etc.; then it occurred to me in vocal motor imagery, 'I now remember what it was about.' Then I set to work to solve it, choosing one pound, two pounds; did not need three because I already had one and two . . .[20] Alway come back to the solving. Four pounds? Yes, I must take that. I do not need five, nor six, because I have four and two; do not need seven—four, two and one. Eight? I need eight, but do not need 10, nor any other up to 16. . . ."

The above introspection indicates that a visual image informed the observer of his former method of solving, which, in turn evoked a remembrance of the statement of the problem and a tendency to resolve it. It is to be noted that the 'short cut,'—the last step in the example given above,—was made in this recall after he had tested the numbers up to eight, while in the first solving he did not generalize until after he had tried every number up to twenty. This 'short cut' in recall was not recognized as the same kind of step that had been discovered during the solving, as his further introspection shows.

Ex. 26. Obs. *B*. ". . . As I looked over the paper at the sitting I found a short cut, which simplified the process of solving it; but I can not find it now. . . ."

The first factor to appear in the recall of this problem, was a visual image of the balance, of the weight (iron), or of the written solution of the problem. In one case the lump of coal (weight) that had been imaged in the solving was the first datum to reappear. Another observer recalled first the diagram that had stood for the solution; eight observers reported that the visual image of the balance appeared first. These images of concrete objects, which the observers had employed during their solving of this problem, now signified to them that they were on the right track, that they had begun to recall the desired problem. These images possessed a definite meaning, which, when attention was directed to them, developed into, or initiated further processes. These processes were very similar to those that are illustrated in Exs. 5, 6, 7, 8, 9, in problem 1. This initial image was clear and stable from the outset; and this fact afforded a feeling of certainty that it was a recall, as is illustrated by the following examples.

[20] At this point came a feeling of doubt. I wondered if I was limited in the number of parts or pieces into which the weight was to be divided. I believe that this consciousness consisted in vocal motor imagery of, 'how many parts is the weight to be divided into?' plus a feeling of pleasantness.

Ex. 27. Obs. *Fs.* [20 sec.] "At first there was an absolute blank; no trace of familiarity; then the word 'divide' vocal motor, and auditory imagery, occurred to me together with a visual image of a cylindrical mass of iron on this white sheet of paper. The cylinder seemed to fall apart into divisions,—20, 10, 5, etc., were present in visual figures. That seemed to involve a consciousness that the weight had been divided into parts,—the latter coming with a good deal of conviction. The essence of that consists in the clearness with which that image appeared,—the whole image was somewhat reduced in size. I was aware of your sitting here, and remembered the awareness of the former situation. Those features were very clear in connection with the certainty that I had begun to recall the correct problem."

Ex. 28. Obs. *Wh.* [5 sec.] "The main points in the problem came out immediately. The word 'weight' appeared in terms of a visual image, exactly the same as I saw at the first sitting. There were the weights used on the scales at home. Recurrences of the visual images I had at the time when I was solving this problem?.[21] I had visual images of the paper I had used in the solution which meant to me that I had been given the problem to divide the weights into one-pound weights, to produce forty one-pound weights."

The following introspections illustrate the extreme activity which sometimes characterizes the act of recall.

Ex. 29. Obs. *B.* ("Recall the weight problem.") [4 sec.] "Farmer and hogs?" ("Introspect.") "Visual image of typically dressed English farmer driving pigs down the road; then a visual image of a typewritten sheet of paper, but could not quite read it. Then, in vocal motor imagery, 'If the hog had weighed twice as much,' repeated several times, but got no farther." [Here, B had evidently been recalling the hog problem.] ("A certain weight to be broken up.") [3 sec.] "Oh, yes. Visual image immediately of a set of weights, for a chemical balance, fitted in a block of wood, silver white, of different sizes. Saw distinctly at last. Then it occurred to me that some object had to be weighed; and that the weights were not of the right size to do the weighing. Even then I was in doubt about details. I think the problem was to take a certain amount of material and weigh it in lots of uniform weight. The structure of this consciousness was a visual image of a druggist who was in the act of weighing, putting up powders in papers, all of equal weight. A feeling of pleasantness and satisfaction that I had succeeded in recalling it. Then I felt that it was another problem I found in the last week. A *Bewusstseinslage* of despair. Had an image of a mass of something that looked like stone, to be weighed, but that flitted past. Then I saw a chemist behind a counter weighing something. The chemical balances were on the counter. This train of images was fleeting and vague. A feeling as of groping for more images to help me solve the problem. . . ."

[21] Saw the weight cut up into different pieces,—cut up into different bundles. Visual image of the word "cut" and a visual image of the figure "40." Then I had a visual image of the problem typewritten on a sheet of paper. Then I saw this much of the statement of the problem "cut up the weights into 40 different pieces," but I did not get any farther.

Ex. 30. Obs. *R.* [3 min.] "I imaged a lump broken into pieces as the other day, then a vocal motor image of 'forty-pound lump.' I almost said 'coal,' but realized it was not coal. The forty pounds to be broken into such pieces that . . . I can't get it. I had a feeling that it was right so far. Rather believe that I failed to solve this problem. This I infer from the feeling of vagueness." [Thinks 2 minutes.] "Many irrelevant images appear; I visualized a pair of scales, visualized '5 to 1,' '3 to 5,' '8 to 5,' '4,' '3,' '2,' etc., meanwhile looking for something which might seem familiar. 'Broken into such parts' came vocal motor. I saw several persons. 'John and Henry' might be given parts. All discarded, not familiar enough. . . . I believe that the problem dealt with an arithmetical progression, which came in the form of a visual image,—a series of digits dimly visualized, '2, 3, 4, 5, 6,' etc." [He had actually set down these figures during the solving.]

C. Main Points and Generalizations from the Recalls of all the Problems.

The question which has concerned us in this investigation may be stated most comprehensively as follows: What mental processes recur in the recall of problems that have been solved but once? We shall here treat these factors separately as nearly as possible in the order of their appearance in the act of recall; and we shall also indicate the relative significance of each of these factors for the act of recall.

1. *Time Required, and Cues Necessary to evoke Recall.* The time necessary for the initial act of recall varied widely for different observers, from almost immediacy to several minutes. If, after an interval of a few minutes, the observer had not obtained even a feeling of familiarity with the object to be recalled, he was given a more specific cue, after which, if still unsuccessful in evoking recall or indications of it, a second cue was presented. In general, those problems which had furnished the more abundant and the more vivid experiences during solving were most readily recalled. The vividness of an experience during the solving may be dependent upon the novelty of the problem, and novelty may therefore be the main cause of ease and rapidity in recall. For example, the 'box problem' was a very ordinary, 'every-day' problem, and it was very slowly recalled by every observer; but the 'spider problem,' which dealt with a strikingly novel situation, was recalled with great rapidity. Those problems in which a physical object, a picture, or a diagram was either presented or imaged, were most easily revived. Problems of the opposite nature such as those of 'war,' 'propositions,' 'linnets,' and 'cramming' were always slow to be recalled; and they always needed many cues to start the recall. In only eleven cases was it necessary to re-present the whole

problem before the observer was able to begin. In such cases he recalled very little of the procedure which he had adopted during the solving. In fact, in several of these cases, the route which had been followed in solving was pursued in the act of recalling but was not recognized.

About half of our recalls were evoked by means of the general instructions: "Recall the first problem you solved;" "Recall another of the problems solved early in our series," etc. We had selected several stereotyped forms for presenting our cues, from which we never departed. Half of these were of a concrete nature and half were abstract. We have made a census of the degree of success or failure which attended the use of each type of cue, and we find no essential difference of evocative value between the two. The abstract cue afforded the observer a wider range of materials to recall and hence our quantitative data are of little value. But we are able to state that the course of recall was very similar in the two cases, except that the abstract cue often aroused several trial images that varied widely, whereas the concrete cue often called forth trial images which varied within narrower bounds. An examination of the introspections which are cited above, shows the results which follow from the employment of various concrete and abstract cues. The employment of a concrete cue should of course be compared with that of an abstract cue in the case of the same observer, as, for example, Exs. 16 and 31.

2. *Attitude and Procedure during Recall.* In the initial stage of a recall, observers vary somewhat widely in their customary attitudes. One observer is very deliberate in his search for the parts of the experience he is attempting to recall. When an image appears, he attends to it very carefully and is in no haste to arouse other images until he has examined this one in detail. During this act he usually discovers many associated images which are present in vague form. He attends to them in a calm and deliberate manner, proceeding in this fashion until finally he has recalled the desired experience. Other observers are very uneasy and struggle very impatiently for a few seconds when one or many images appear which arouse still greater excitement. Their goal is to evoke the problem in all its details at once; and it often happens that many of the images are passed over slightingly, which, if attended to more carefully, would undoubtedly establish a firmer route of recall.

It is very interesting to note the many goals which the observer constructs for his procedure in the various recalls.

The deliberate observer mentioned above sets about very often to find a significance for a single unimportant datum that appears, and in so doing wastes his time. The hurried observer is apt to rush on beyond some important images and lose himself in minor details, consequently is unable to formulate the problem or its solution without reviewing and reorganizing his objects of recall. The following instances show typical attitudes during recall: Exs. 10, 14, 24, 27, 30, 31. (Read in this connection the description of the characteristics of the appearance of the images of recall, p. 72.)

In most cases of recalling after a month's interval there is first a period of searching which continues for only a few seconds, and is immediately followed by a more or less profuse succession of images. When this second stage is reached the observer has usually ceased his search; images now flood in upon him. His attention is commanded by them, rather than they by his attention, as was the case in the first stage mentioned. Many deviations are found, but in a majority of cases the above two stages are present.

3. *The Structure of Recalled Experiences.* a. The Feeling of Familiarity. As has been shown above in the statement of the results of the individual problems there sometimes occurs a feeling of familiarity, a feeling that the earlier experience is just about to be revived. This is not an unusual phenomenon in every-day life; its significance for recall seems to consist, for the most part, in the fact that it gives encouragement and incites one to grope and search for the datum which is almost within one's reach. The number of attempts to recall, in which this feeling of familiarity preceded, is relatively very small in our experiment. Exs. 3 and 4 show the general characteristics of such an experience.

b. The Image. In ninety-five per cent. of our recalls a visual image was the first definite factor to appear. This characteristic may be seen to be prominent by glancing over the introspections quoted above. It is surprising to discover that both the solving and the recalling of such a variety of problems is mediated by visual imagery. Although tactual, temperature, kinaesthetic, auditory and vocal motor images had been employed in the solving, many of the problems were recalled in terms of visual imagery, such as, images of one's self writing, of sitting in a thoughtful posture, etc. (See Ex. 9, p. 51, Exs. 15, 16, pp 56f.) However, in the cases of those problems which had to do less with concrete objects than with turns of language or with hidden meanings or

sentiments, verbal imagery,—auditory, vocal motor, and visual,—appeared more often than elsewhere.

On account of lack of space we do not include a table showing quantitative data regarding the various kinds of imagery that appear in recall, for each of the three modes of initial presentation of the problems that we employed. This correlation is of little value, because in the solving of our problems the mental processes are far more numerous and more significant to the progress in the solution than is the mode of presentation. In only a third of the cases of recall did the observer evoke a remembrance of the mode of presentation, visual (print or script), or auditory; and this remembrance was usually the reappearance of a single word, or visual form, as *e.g.,* the auditory word 'six,' or the typewritten numbers 6, 6, 4, or the circles arranged in the form of a square, as in problem 1.

c. Characteristics of the Appearance of the Images of Recall. In a great majority of the recalls, the images first appeared in a rather vague and schematic form, no matter to what modality they belonged. Tactual remembrances appeared in a series of not definitely localizable images of tactual impressions upon the forehead. The kinaesthetic imagery of moving the lines in the 'dot problem' (see Ex. 9, p. 51, and p. 56, the 'ring problem'), never became clear and definite. Vocal-motor and auditory images often came in a vague and disconnected manner. Observer *W,* who reports more auditory memory than any other observer, often images words but these so indefinitely that for some time they cannot be spoken nor can their meaning be given. Many examples of our recalls, which for want of space we cannot include here, show that although the images first appear very vaguely, they gradually develop, and become definite and meaningful. (See Exs. 22 and 25, pp. 61 and 67, Ex. 27, p. 68, etc.)

The images that come readily and are clear at their first appearance are usually endowed with particular meaning. For example, when the observer without any other clue than "Recall another problem" had a clear visual image of a farmer driving hogs down a country road, or of a milkman with a number of milk cans, or of a schematic cross-section of the earth, he had at the same time a meaning for each. A farmer driving hogs has almost always been associated with going to market; this association was therefore immediately reinstated, and the image was endowed with meaning,—a problem about a farmer taking hogs to market. Then came the association (not a new one at all) that hogs had been sold, and that the problem had to do with the weight or the price. Now the two questions which are the most

usual ones in every sale of hogs are: What is their weight? and what is the price per pound? The word weight as it occurred in a visual, an auditory, or a vocal-motor image, or was actually spoken, was very familiar. He remembered that the problem was about weight; and, at this point, he saw his diagram or his algebraic solution that he had made during the solving; or the vocal-motor, auditory, or visual image "half" was recalled. The problem and its solution were now almost completely revived. A few other words or symbols appeared,—such as, 'score,' '7 score,' '140,' '280,' or 'half its weight,'—and these soon found their proper setting, and this completed the recall. Such an act of recall was often accomplished in a few seconds. In certain cases, however, the processes of recall continued for several minutes.

A very clear example of growth in definiteness, in detail and in meaning is found in every observer's recall of the 'density' or 'boat problem.' When the cue "Recall the density problem" was given, there appeared either the visual image of the boat (problem *22*) that they had imaged in solving this problem, or a schematic visual image of a cross-section of the earth (problem *24*). Whichever came was satisfactory and seemed to the observer to be appropriate. For instance, he visualized the boat passing from fresh to salt water, and rising just as it did in the initial apprehension of the problem. Several observers stated the problem at this point, by translating their visual imagery into words: 'If a boat passes from fresh to salt water, will it rise or sink in the water?' But they did not rest here, for this statement seemed too simple when it had been expressed and completely envisaged. Immediately afterwards there appeared a visual image of the water-line, or a visual, auditory or vocal-motor image of the word 'water-line.' This signified that the problem had to do with the water-line; hence they stated the question in some such manner as this: 'Will the water-line rise or fall when a boat passes from fresh to salt water?' Hereupon they visualized the two water-lines,—first, the line of intersection of the surface of the fresh water with the boat, and second, that of the boat with salt water. In these images they perceived the answer that 'The water-line falls.' No one remembered the answer in a purely verbal form. They all recalled it in imagery of a concrete visual sort. Every observer first recalled the solution of the previous day; six recalled no more; while four saw again the old predicament that had baffled them, the predicament that after they had attended to the visual image

of a fixed water-line painted on the hull, they conceived a second meaning of water-line,—the movable line of contact of the water upon the hull (a series of visual images of the boat showing the shifting line). This movable line they saw lower on the side of the boat with the rise of the boat; and if that be the correct meaning of water-line, then the answer is that the water-line falls. The meaning here was contained in the images. Without them there was no meaning; and beyond them and the context associated with them no observer could report any meaning. This whole experience was as clearly a revival of the original perception as can be found. A typical example of the development of images and meaning is given below.

Ex. 31. Obs. *G.* ("Recall the density problem.") [14 sec.] "I repeated the word; then had a visual image of an oil can and of the word oil, but dismissed them. Then one of oil and water in the can. This was all over before I could shut them off. Then a vague visual image of a ship, on the ocean. Then a question, in vocal-motor terms: 'Would it float higher on salt water than on fresh?' A visual image of the ship rising as it went into salt water. I was then aware that I was recalling the right problem. . . ." "I saw water-line on the ship going up with the ship. Then I saw the water-line going down while the ship rose. Here I had a faint auditory image of myself asking you, during the former sitting, 'What is meant by water-line,—its definition?' I now had a meaning of water-line, in visual imagery from the beginning. I saw first the ship going up and I just saw the line on the boat going up. Then I looked at the water-line going down, *i. e.*, the contact of the surface of the water on the hull going down as the boat rose. This was the final meaning it seems; it remains stable."

The above introspection shows clearly the course of development from a vague schematic image to one which is rich in meaning; it illustrates the trial images which occur in the beginning before the accepted recalled image appeared; and it shows that the meaning is bound up with or contained in the images. The permanence of attention upon the last-mentioned of the images signified its correctness. This is a typical example of the attitude of certainty which is induced by stability, by clearness of the image, or by lack of other images; this attitude is frequently allied with feelings of satisfaction, and the like. It is interesting to note that no characteristic difference exists between the certainty which occurs in recall and that which occurs in the first solving. (For illustrations of this see Exs. 10, 13, 14, 24, 27, 31.)

As has been shown in Exs. 12, 15, 18, 24, 25, 30, 31, which are typical of the other problems, there has been much trial imagery. Many gaps have been filled in by a process which

is in nature very similar to creative imagination. In accordance with our definition (p. 46), we have in many instances, called this process reasoning. The observer often infers that more of the problem is yet to be recalled, because the bits of experience that he has revived constitute neither a problem nor its solution; or because what he has recalled may appear to be much more simple than any other problem that he remembers. Consequently he constructs and fills in the parts necessary to formulate a probable problem. He often writes down this material so that he may test its familiarity to the eye; he may say it over to test it by the ear, or by its vocal-motor 'feel' (the 'feeling' of saying it). These aids often bring the recognition or rejection very soon; and in many instances they promote further recall. (See Exs. 12, 19, etc.; also see discussion, pp. 50 and 73.)

Throughout the various problems that we have treated in more or less detail we have, here and there, seen examples of an abundance and even a profusion of images (Exs. 6, 7, 8, pp. 50f.; 12, p. 53; 15, p. 56; 22, p. 61; 24, p. 63). Often during a few seconds of recall, many more images had been present than could be stated. At times every observer exhibited this characteristic; however, it was more common for some than for others. It seemed to depend, in a great measure, upon the condition of the observer. One who is least likely to be excited under trying conditions and who always deliberates, seldom found so many rapidly fleeting images that he was unable to describe them. Several of our observers who are readily aroused or easily disturbed in ordinary activities often become overwhelmed with a multitude of fleeting images. These may be not only of one sensory modality but of several at the same time. Ex. 6, p. 50, Ex. 24, p. 63, Ex. 30, p. 69, as well as the following illustrate this point.

Ex. 32. Obs. *Hn.* ('Recall the ring problem.') [22 sec.] "I had a clear visual image of the ring,—the motor twist of the solution, the methods of trying, etc.,—all about equally vivid and simultaneous."

Ex. 33. Obs. *M.* ("Recall the problem next in order.") "I recall the whole experience, images of this room and of the general situation. It was a dark, warm and cloudy day and the hall leading to this room was dimly lighted. I saw the pieces of paper and myself drawing the diagrams. These came in exceedingly rapid succession. . . ."

If an observer had remembered very little of an experience, his recall was in most cases extremely slow and his images were faint and few. (Exs. 3 and 4.) This is in

contrast to the recall of an experience which had been almost entirely remembered, in which case the images were many, vivid, and stable, as Ex. 32 shows.

Let us summarize here the essential characteristics of the recall. When one is able to recall a series of data which corroborate one another, feelings of familiarity, of certainty and of reality are aroused, although the images be in some cases fleeting and vague. The observer then 'knows that he knows.' In other cases a few very vivid images also afford a high degree of certainty to the observer. Such images are recurrent or stable; and if they do not conflict with one another, they signify to the observer that his recall is correct. No single datum of recall is sufficient to enable an observer to say that a given mental experience is a recall. The series of corroborative images (or the context) which gives rise to feelings of familiarity, and the like, is the prime factor in distinguishing an experience as one of recall.

d. Affective Processes. A number of our problems were worded in such a manner that the observer became suspicious of a 'catch' and attacked the solution only after a very careful reading. He often felt that he should be able to solve a problem creditably, yet he could not do so for some time. Consequently various 'feelings' of doubt, of uncertainty regarding his own ability to solve the problem, of chagrin and even of despair, arose before the dawn of successful operations. These were frequently followed by 'feelings' of buoyancy, triumph and satisfaction. In only a very few instances did the observer, after a month or more, recall that any of these feelings had been present. Intensive feelings of chagrin, which had arisen when the observer was reproved for disobeying instructions, were never recalled even though the other components of the experience were readily reinstated. None of our three score recalls of affective processes were evoked except after or together with images of parts of the former experience. We infer that they are of relatively little moment in the act of recall. Ex. 21, p. 61, and the following examples show the recall of affective processes connected with imagery.

Ex. 34. Obs. *Wld.* ("Recall the second problem.") [This is the 'train problem.' 25 sec.] "I believe that the cars and shunting came second. I had a visual image of a T, then the whole situation, where I sat, where you sat, how I solved the train problem, the diagram, all very vaguely. A real track was seen in visual imagery. As I described it, I became more certain that it was the next problem after the T; and there came a faint remembrance of the feeling of novelty that I had had at the time of solving."

Ex. 35. *Ck.* [Recall of the Uncle and Nephew problem.] ". . . I involuntarily began to reconstruct the scene and at the close of this visual recall of the room, the day, and the problem, I recalled the original satisfaction that had attended the solving. This clear, affective toning was a part of the recall of the earlier experience. . . ."

For some observers various attitudes of helplessness, discouragement, and despair frequently arise during the initial stage of recall when the experience had been almost completely forgotten. These are not common, nor is there any regularity in their appearance.

e. The Recall of Processes of Reasoning. 'Short cuts' during the solving were discovered by every observer, but they were seldom recalled. In introspection 25 the 'short cut' which had been made during the solving was hit upon at an earlier stage in the re-solving; but it was not recognized as the step which had been taken during the initial solving. Ex. 12 and 13 also show that the processes which we term reasoning and which had been present during the solving were not recalled. The recall of 'short cuts' is best illustrated in Ex. 28, and in the counter problem, number 8. Nearly every observer reported a visual remembrance of the act of moving a series of counters by means of a single move of the hand; but they counted it as many moves, because many counters had been included in the single movement. The visual image of the pile of counters, of the hand shifting them by a single movement, and of the numbers were the contents of this remembrance of the 'short cut.'

Resumé and Critique

The eighteen individuals who served as observers were drawn from China, from Japan and from widely separated regions of the United States and Canada. The striking similarity in mental content and in mental procedure which our investigation has revealed can scarcely be referred to similarities of social or educational environment and training. Our results differ widely from those of the German and French investigators who report the discovery of non-imaginal elements that stand alone in consciousness; we have found that no datum was ever recalled without images. These images, however, frequently embodied very intricate features. For example, the image of the weights which constituted the solution in the weight problem, of the water-line in the schematic image of the side of the boat, which constituted the crux of problem 22, and of the path followed by the spider (problem 12) were rich in meaning, and they furnished the most essen-

tial logical relations of these problems. Our results do not show a single remembrance of relations,—such as perpendicularity, contrast, proximity, etc.,—except in connection with some sort of imagery, vague and schematic though it frequently was.

Michotte and Ransy confined their investigation to the remembrance of related words and phrases. Their observers did not report what relations had been perceived during the initial observation; and consequently they could not guarantee the recall of any, although they reported the recall of many relations as non-imaginal elements in consciousness. The cue which these investigators employed to initiate the act of recall was one of the words of the pair between which the relation was supposed to have been perceived during the initial observation; and for this reason it seems probable that the alleged recalls of relations were perceptions of relations within the perception or the conception of the stimulus-word which was given to evoke recall. Hence their findings do not seem to justify the statement that one may recall relations which are the sole content of consciousness, and which are, therefore, elemental in nature. If the observers had been asked to recall the whole experience without obtaining a single cue, undoubtedly their report that recalled relations stand alone in consciousness or exist only in connection with imaginal components would have been more convincing.

Bühler reports that his investigation also reveals the existence of a consciousness of relation wherein no imaginal content is present. His investigation is clearly open to the same criticism that we have urged against the above investigation. Schwiete found that feelings of familiarity constitute the sole structure of certain concepts for some observers. But here again one has no guarantee that the concept which he is attempting to ideate is completely aroused apart from the perception of his stimulus-word. The feelings of familiarity that he found may be very like those that we have noted in the stage of recall which precedes the appearance of definite features. A second essential difference between Schwiete's investigation and our own consists in the fact that his materials were more abstract than ours, since he employed general ideas and relations that had been habitually perceived while those that we employed had been perceived only once or a few times at most; because of this fact one might expect that Schwiete's method would arouse less concrete contents than our own experiments revealed.

Several investigators have mentioned the predominance of

the visual image in recall; but none have found so high a percentage of the components to be visual imagery as our results indicate. Our results also show that vague and schematic visual imagery is very frequent, especially during the earlier stage of the act of recall. The cause of this extreme frequency of visual imagery is not apparent from the present investigation; but it seems probable that the extreme unfamiliarity and novelty of a majority of our problems was effective in producing this condition. Our procedure demanded a very thorough ideation of the data of recall because our problems were rather complicated and involve very delicate shades of meaning; we may infer that this more complete ideation may account in part for the striking predominance of visual images in the present investigation.

V. Summary

1. The act of voluntary recall is usually characterized by two stages. The initial stage consists of an *Aufgabe* to recall and an act of 'searching.' It may continue for only an instant or for several minutes. When the desired datum is readily recalled, the consciousness of activity may be of slight intensity; in extreme instances it is wholly absent. But in cases of difficult or hesitant recall, the act of searching may be conscious, voluntary and long-continued. During this initial stage the observer adopts either a relatively active attitude and procedure, which consists in a vigorous groping and an attentive searching for missing data, often calling up all sorts of images and reviewing them to determine whether or not they are the missing data or hoping that they may cause the arousal of the desired past experience, or a relatively passive attitude and procedure which consists in calmly, but expectantly awaiting the images of recall. Either procedure may be carried through in an unsystematic fashion,—as a planless examination of images which are already present to consciousness, and an equally planless groping for images which seem to lie just beyond one's reach; but in numerous instances, the observer proceeds in a deliberate, methodical and systematic fashion. Occasionally during the long continued search, definite and intensive *Bewusstseinslagen* of satisfaction, buoyancy, doubt, helplessness, despair, or others, enter consciousness. Their arousal is apparently due to the state (psychical and physiological) of the observer. When he is especially active and free from fatigue despair never arises; but the opposite physical condition is almost sure to

give rise to feelings of both doubt and despair. None of these have been sufficiently regular or definite to guarantee our attaching any special significance to them.

2. A second stage, is characterized by imagery of various sorts, present in more or less profusion and in greater or less degree of intensity, or vividness. In a majority of the cases, vague schematic images of general or total situations, are first to appear. Their meaning is vague and becomes clear only with their growth in definiteness. One part of the image clears up, then another. Other images reinforce these, until the initial experience may be reinstated in almost complete form. The fact that these images are stable or recurrent, and the additional fact that they corroborate one another, though they may be incoherent and full of lacunae, arouse the 'feeling of familiarity' and signify to the observer that his procedure is one of recall and not one of creative imagination.

3. Our results show that the concrete objects or words of the initial perception, and the images employed during the act of solving problems are very much more likely to appear (in the form of images) in recall than are the 'short cuts' or processes of reasoning. The concrete images usher in meanings, and associations are made which then constitute the awareness of relation. The description of the imagery in the 'hog problem' (p. 72), and the Exs. 25 and 31 (pp. 67 and 74), illustrate this dawn of meaning and awareness of relations. These latter are clearly recalled in terms of coherent imagery. In other words, *content* was readily recalled while *act* was almost never recalled.

4. Very few of the unsuccessful trials in the solving are definitely recalled. The first unsuccessful trial that has been made is usually the only one ever recalled. This initial trial appears in about a third of the recalls; but the observer seldom fails to recognize that it had previously been present to consciousness as an error. The remembrance of other errors is present in an extremely vague and schematic fashion, *e.g.*, a visual image of splotches on the paper used in solving, etc.

5. The cues of an abstract and those of a concrete nature lead to approximately the same results.

6. Most observers tend to recall more data in visual imagery than was present in visual imagery during the solving. Some few observers show a slight tendency to recall in vocal-motor terms certain data which have been present during the initial experience in other modalities of sensory imagery. One observer employs in recall far more auditory imagery

than any other observer, and yet approximately only ten per cent. of her images during the act of solving are auditory.

7. The problems that deal with logical fallacies, and sentiments, are more frequently recalled in verbal imagery than are other problems. Here the verbal imagery constitutes about one-third of the content of recall; while in the recall of other problems, it constitutes less than one-tenth of the content.

8. The problems which present most novel features to the observer, as for example, 'the dot,' 'the spider,' and 'the density' problems, are recalled most predominantly in visual imagery.

9. The visual image is the strikingly dominant feature of recall. Feelings of familiarity and of satisfaction, pauses of attention and the like, are seldom recalled; they are never recalled except in connection with imagery. No recall of an affective state, of an attitude (*Bewusstseinslage*), of an *Aufgabe,* of steps in reasoning, or of a concept has been reported, except with images. From an actual count of factors present in the recall of ten of our problems, we estimate that our investigation embraces approximately 200,000 images but only 100 other components; and these latter are for the most part of an affective character. Of all our introspective data, about ninety per cent. are visual images.